IMPLICATIONS OF THE UPDATED ADAPT-VPA ASSESSMENTS FOR THE DYNAMICS OF MINKE WHALES IN AREAS IV AND V

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ABSTRACT
A stock-recruitment model of the Pella-Tomlinson form is fit to recruitment and adult female abundance estimates from the Base Case ADAPT-VPA assessment of Areas IV+V combined. The trends of these plots require the assumption that minke whale carrying capacity first increased, then later decreased during the 20th Century. An initial attempt at this fit suggests that this carrying capacity increased about five fold from 1930 to the mid-1960s, and then decreased again by about half. MSYR is estimated at 4.0% for this model.

INTRODUCTION
This paper focuses on the results of the Base Case assessment of Areas IV and V combined that is presented in Mori and Butterworth (2005), with a view to improved understanding of the dynamics of the minke whales in this region.

Fig. 1a plots the recruitment \( N_{y,1} \) time series estimated by this Base Case assessment. Fig. 1b shows the associated stock recruitment plot: \( N_y^A \ vs \ N_{y-1}^f \), where the “adult” (reproductive) population is taken to be:

\[
N_y^A = \sum_{a=1}^{45} N_{y,a} \tag{1}
\]

and the number of adult females \( N_y^f = 0.5 \cdot N_y^A \), i.e. an age at first-parturition of 7 is assumed. This plot includes a “replacement line”; this is the line where recruitment exactly balances deaths from natural mortality, i.e. for a point on this line, population size is maintained unchanged for the following year in the absence of harvesting.

For a conventional stock-recruitment relationship, recruitment levels would lie above the replacement line at low adult female population sizes, but (in the absence of harvesting) as carrying capacity \( K \) was approached, recruitments would cluster around a stable equilibrium point on the replacement line. This, however, is NOT the behaviour evident in Fig. 1b. The recruitment series does approach the replacement line (in about 1973), but thereafter drops below this line as both recruitment and the number of reproductive animals decrease.

Such behaviour requires not only the conventional assumption of an increase in the minke whale carrying capacity \( K \) in the mid-20th Century as the then unharvested minkes increased, presumably in response to the depletion of other krill-feeding Antarctic baleen whale species through overharvesting, but also a subsequent decline in \( K \) (a
result also demonstrated earlier by Butterworth and Punt, 1999). Possibly this could be a response to recent recoveries of these other species.

This paper attempts an initial model fit to the stock-recruitment information in Fig. 1b, to assess to what quantitative extent changes in $K$ are required.

THE RECRUITMENT MODEL

Recruitment is assumed to follow a Pella-Tomlinson form:

$$N_{y+1,1} = \lambda \cdot N_y^f \left[ 1 + A \left( 1 - \left( \frac{N_y^f}{K_y^f} \right)^z \right) \right]$$  \hspace{1cm} (2)

where

- $N_{y,1}$ is the recruitment (1-year-olds) in year $y$,
- $\lambda$ is the combined pregnancy and first year survival rate when the population is at carrying capacity,
- $N_y^f$ is the number of adult (past the age of first parturition) females, taken to be given by $0.5 \sum_{a=5}^{45} N_{y,a} = 0.5 N_y^A$ (i.e. we assume equal numbers of males and females),
- $A$ is the resilience parameter (related to MSYR),
- $K_y^f$ is the carrying capacity for adult females, which may change over time, and
- $z$ is the degree of compensation parameter, which we set here at 2.39.

When $N_y^f = K_y^f$, the recruitment must equal the number of 1+ whales dying annually as a result of natural mortality, i.e:

$$\lambda \cdot K_y^f = K^{1e} \left( 1 - e^{-M} \right)$$  \hspace{1cm} (3)

(ignoring for simplicity the small correction that accounts for an assumed infinite natural morality from age 45).

Further, expressions for unexploited equilibrium numbers at age values yield:

$$\frac{K^{1e}}{K^f} = \frac{\sum_{a=5}^{45} e^{-M_a} \alpha}{0.5 \sum_{a=5}^{45} e^{-M_a}} = \mu$$  \hspace{1cm} (4)

where $\mu$ can be computed given the value of $M$ (0.068 for the assessment to be considered here).

Thus equation (2) can be rewritten:

$$N_{y+1,1} = \mu \left( 1 - e^{-M} \right) N_y^f \left[ 1 + A \left( 1 - \left( \frac{N_y^f}{K_y^f} \right)^{2.39} \right) \right]$$  \hspace{1cm} (5)

The unknown parameters of this model are $A$ and the parameters describing $K$ and its temporal variation. These
are then estimated by minimizing:

\[ SS = \sum_y \left( \ln \left( \frac{N^{\text{obs}}_{y,1}}{N_{y,1}} \right) - \ln \left( N_{y,1}^{\text{model}} \right) \right)^2 \]  

(6)

where

- \( N^{\text{obs}}_{y,1} \) is the “observed” recruitment for year \( y \) from the Base Case assessment for Areas IV+V combined, as shown in Fig. 1a, and
- \( N_{y,1}^{\text{model}} \) is the recruitment for year \( y \) predicted by the model of equation (5).

**RESULTS**

Thus far, the following functional form for \( K_y^f \) has been considered (see Fig. 2):

\[
K_y^f = \begin{cases} 
K_1^f & y \leq y_1 \\
K_1^f + \left( \frac{K_2^f - K_1^f}{y_2 - y_1} \right) (y - y_1)^y & y_1 + 1 \leq y \leq y_2 \\
K_2^f + \left( \frac{K_3^f - K_2^f}{y_3 - y_2} \right) (y - y_2)^y & y_2 + 1 \leq y \leq y_3 \\
K_3^f & y_3 + 1 \leq y 
\end{cases}
\]  

(7)

with the following choices made for the “change” years: \( y_1 = 1930, \ y_2 = 1966 \) and \( y_3 = 2000 \).

The results of the fitting process yield: \( A = 1.28, \ \gamma = 1.530, \ K_1^f = 36639, \ K_2^f = 151455 \) and \( K_3^f = 69089 \), suggesting that carrying capacity originally increased about five-fold, but has since approximately halved.

The model fit itself is shown for recruitment vs year in Fig. 3a, and for the stock recruitment plot in Fig. 3b. The estimated trends in carrying capacity are shown in Fig. 4.

**DISCUSSION**

The fit of the model to the recruitment and stock-recruit plots in Fig. 3, while showing qualitative consistency, clearly manifests some systematic deviations. Further work will seek to modify the functional form assumed for \( K_y^f \) to achieve a better fit.

The estimation of the value of resilience \( A \) in turn allows an estimate of \( MSYR = F_{MSY} \) to be effected. The result is:

\[ MSYR_{1+} = 0.040 \]  

(8)

with a corresponding adult-female population size in terms of the corresponding carrying capacity:

\[ MSY_{\text{mat}} = N^{f} (F_{MSY})^{k_f} = 0.548 \]  

(9)
The model of equation (5) could be used to project the abundance trajectories estimated in Mori and Butterworth (2005) forward in time under alternative future catch levels. Indeed, the stock-recruitment model here could be incorporated in the ADAPT-VPA assessment process itself, then also providing an improved approach to the process used by Mori and Butterworth (2005) to “adjust” earlier estimates of abundance for “missing” cohorts.

REFERENCES

Figure 1. The “observed” recruitment time trend and stock-recruit plot for the Base Case Areas IV+V combined ADAPT-VPA assessment of Mori and Butterworth (2005).

Figure 2. The functional form assumed for the change over time in carrying capacity of adult females.
Figure 3. Fit of the model of equation (5) to the recruitment trend and stock-recruit plot for the Base Case Areas IV+V combined ADAPT-VPA assessment.

Figure 4. The estimated trend in carrying capacity of adult females over time.